FRICTION TORQUE IN A ROTATING FIELD AND THE MAGNETORHEOLOGICAL EFFECT IN COLLOIDAL FERROMAGNETICS

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It is shown on the basis of theory and experiment that interaction between particles produces a friction torque in a rotating field or a force in a stationary field, which does not depend on the rotational frequency or on the rate of shear, respectively.

The idea of Rosensweig [1] of using "magnetic fluids," i.e., colloidal ferromagnetics for a direct conversion of heat into mechanical energy or for other purposes has been recently followed up by several articles [2-5] which explain the phenomena revealed in experiments [6-9]: by an increased viscosity of these fluids in a magnetic field (magnetorheological effect) [2, 3] and by the appearance of a torque in a rotating field which will rotate the vessel containing such a fluid about the field axis [4, 5, 9].

The authors of [2-5] do not connect these two effects, although both are similar in many aspects: each is explained by the generation of microvortices around particles and either a rotation of particles by an external magnetic rotating field [4, 5] or their streamlining by the fluid with a velocity gradient q in a stationary field resisting such a rotation. In the latter case, the velocity gradient produces a circulation of the fluid around a particle. In both cases the rotation of particles relative to the medium (or of the medium relative to particles) results in an additional dissipation of energy and the appearance of macroforces (friction torque in a rotating field or an increased viscosity).

Such an approach does not take into account the interaction between particles, which in a magnetic field leads to the formation of particle chains [10] oriented in the direction of the field and to a change in the magnetic susceptibility [11].

In the steady-state mode the chains rotate with the field at its angular frequency ω. On every particle there acts a viscous friction force approximately equal to the Stokes force $F_i = B\omega r_i$, with B denoting a certain coefficient and r_i denoting the distance from the i-th particle to the center of the chain.

If r is the distance between neighboring particles, which depends on the magnitude of superficial repulsion forces between the particles [12], then $r_i = ir$ if there is an odd number of particles and $r_i = (i-0.5)r$ if there is an even number of particles in the chain.

The shearing force at the chain center is $F(\nu) = B\omega \sum_{i=1}^{\nu/2} r_i$. The results of summation for an even and an odd ν are almost the same, except for the correction term $-1/\nu^2$, and equal to

$$F(\nu) = \frac{1}{8} Br\omega\nu^2. \quad (1)$$

In an analogous manner one finds the friction torque of a chain $M_1 = 2\sum_{i=1}^{\nu/2} F_i r_i$. Here $M_1 = (1/12)B\omega r^2\nu^3$, accurately within $-1/\nu$, and the specific (per unit volume) friction torque

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$$M \equiv M_1 \frac{n}{\nu} = \frac{1}{12} nB\omega r^2 \nu^2, \tag{2}$$

where n is the number of particles and n/ν is the number of particles per unit volume.

The equilibrium number of particles in a chain is determined by the strength of the chain. Under surface forces which act for a much shorter duration than dipole forces and in a strong external field ($H \gg \overline{m}/r^3$) a chain is subject to

$$F(\nu) \simeq \frac{3\overline{m}^2}{r^4}, \tag{3}$$

where $\overline{m}$ denotes the mean magnetic moment of a particle. In the case of single-domain particles $\overline{m} = mL(mH/kT)$, where L is the Langevin function and m is the constant moment of a particle. Combining (1), (2), and (3), we obtain

$$M \simeq 2n \frac{\overline{m}^2}{r^3}, \tag{4}$$

i.e., the friction torque due to interaction between particles is numerically equal to the specific energy of magnetic interaction between such particles and does not depend on B, ω, and ν. The total friction torque

$$M_T = M + M_\omega \tag{5}$$

includes torque M which is due to the intrinsic rotation of particles and which is a function of B as well as of ω. According to [9], for instance, $M_\omega = p\varphi\eta\omega$ (p is the particle form factor, φ is the volume concentration of particles in the dispersion, and η is the viscosity of the medium).

It follows from (4) and (5) that $M = \lim_{\omega \to 0} M_T \neq 0$, which explains the dependence of the effect on the angular frequency of the rotating field. As the radius of action of superficial repulsion forces decreases, the interaction torque M increases sharply and, at low frequencies ($\omega \lesssim 100$ sec^{-1}), exceeds M_ω by a few orders of magnitude – the torque M_ω having been considered at all only in [2-9] anyway. At $n = 10^{16}$ and $m \simeq 2 \cdot 10^{-16}$ erg/Oe, which are usual values for colloidal ferromagnetics, the equilibrium distance between particles is $r = 3 \cdot 10^{-6}$ cm [11], $\eta = 10^{-2}$ P, and $\varphi = mn/I_S' \simeq 0.01$ (where $I_S' \simeq 200$ G is the saturation magnetization of high-dispersion magnetite [13]), in a field of intensity H = 1000 Oe and with frequency $\omega = 10^3$ sec^{-1}

$$M_T = 2 \cdot 10^{16} \frac{4 \cdot 10^{-32}}{27 \cdot 10^{-18}} L^2 + 6 \cdot 0.01 \cdot 10^{-2} \cdot 10^3;$$

$$\frac{mH}{kT} = \frac{2 \cdot 10^{-16} \cdot 10^3}{4 \cdot 10^{-14}} = 5; \; L \simeq 1 - 0.2 = 0.8; \; L^2 = 0.64; \; M_T = 20 + 0.6.$$

The first term represents the interaction effect and the second term represents the contribution of intrinsic particle rotation. The latter term may be somewhat larger, owing to the solvation of particles or the presence of adsorption and ionic layers [7, 8].

In a simple shear flow the velocity gradient $q \equiv \omega$ and, in a magnetic field perpendicular to the slip planes, the chains are subject to the same forces as in a rotating field. Under these conditions the friction force due to energy dissipation in the chains is numerically equal to M:

$$F_S \equiv M \simeq 2n \frac{\overline{m}^2}{r^3}. \tag{6}$$

Analogously to (5), the total force

$$F_T = F_0 + F_S + F_E \tag{7}$$

or $\eta_T = \eta_0 + \eta_S + \eta_E$. Here $\eta_S = F_S/q$ is the structural viscosity, $\eta_0 + \eta_E = \eta_0(1 + p\varphi + \ldots)$ is the Einsteinian viscosity in the field [2, 3, 14], i.e., the viscosity of a completely broken down structure [15], and η_0 is the viscosity of the medium. If the correction term $-1/\nu$ is not omitted in (2), then $F_S = M \sim \nu^2(1-1/\nu)$ and at large gradients $\nu \to 1$, $F_S \to 0$, and thus $\eta_T \to \eta_0 + \eta_E$. It is easy to see that expression (7) agrees with the Shvedov–Bingham equation. The inverse proportionality between η_S and q according to Eq. (6) has been confirmed by experimental data on the electroviscosity effect [16, 17], which is analogous to the

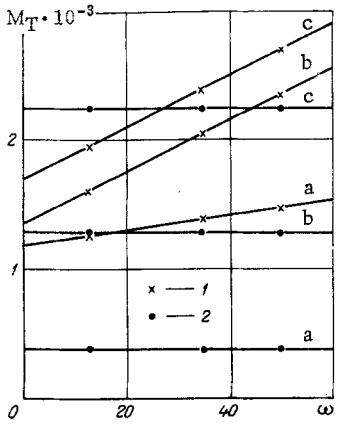

Fig. 1

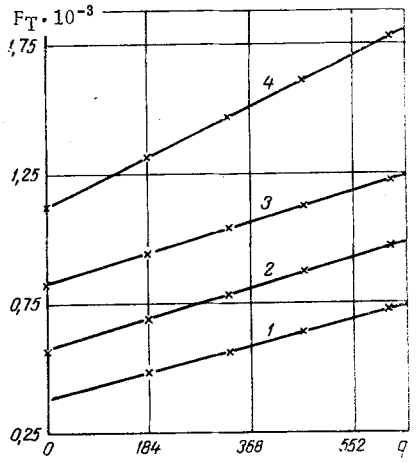

Fig. 2

Fig. 1. Friction torque M_T (dyne/cm^2) as a function of the angular frequency ω (sec^{-1}) and of the field intensity H (Oe), for a barium hexaferrite suspension (φ = 0.115 cm^3/cm^3): 1) in spindle oil; 2) in octane with a 0.3% trace of oleinic acid; a) at 200 Oe; b) at 400 Oe; c) at 800 Oe.

Fig. 2. Friction force F_T (dyne/cm^2) of barium hexaferrite in spindle oil (φ = 0.115 cm^3/cm^3), as a function of the velocity gradient q (sec^{-1}) and of the field intensity H (Oe): 1) H = 0; 2) 220 Oe; 3) 430 Oe; 4) 850 Oe.

magnetoviscosity effect, and by measurements of F_T and M_T on suspensions of magnetic materials (Figs. 1, 2, and 3). The friction torque in a rotating magnetic field was measured by the twist angle of a cylindrical vial containing colloidal fluid and suspended from an elastic string. The data in [18] on the performance of magnetic couplings are also in accord with formula (6). Evidently, a relation similar to (6) is valid, i.e., the structural component of viscous friction forces is equal to the specific energy of particle interaction also in the case of three-dimensional structures, regardless of the nature of the interaction forces.

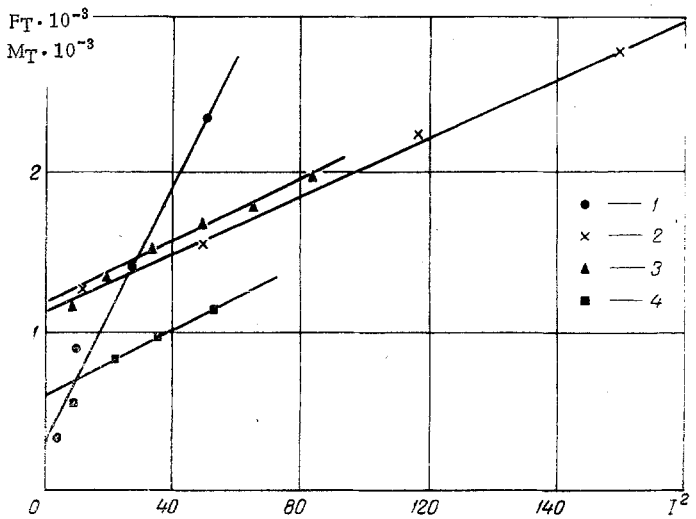

Fig. 3. Friction torque M_T (dyne/cm^2) (1, 2, 3) and friction force F_T (dyne/cm^2) (4) as functions of the square of the magnetization I^2 (G)^2 for barium hexaferrite suspension: 1) in octane φ = 0.182 cm^3/cm^3; 2) in octane with a 0.3% trace of oleinic acid, φ = 0.115 cm^3/cm^3; 3, 4) in spindle oil, φ = 0.115 cm^3/cm^3.

If the results of measurements are represented in terms of an $M_T = M(I^2)$ relation, then an intercept $M(0) \neq 0$ will be noted on the axis of ordinates (Fig. 3). This means that bonds between particles are produced spontaneously too, in the absence of an external field and without residual induction. Particle conglomerations have a complex structure outside the field, but within the field they straighten out into chains which contribute to the total torque M_T independently of I. In some of these chains the bond between particles is stronger, apparently, owing to the action of molecular coupling forces. These stronger chain aggregates, unlike the others, retain their size (the number of particles in a chain) according to (1), i.e., behave like rigid elongated particles and they contribute to the total torque M_T an amount proportional to $\eta\omega$. This, as well as the solvation of particles [6-8] explains that $(1/\eta)(dM_T/d\omega)$ is usually larger than $p\varphi$ at a given concentration of particles φ. Instead of (5), therefore, one may write

$$M_T = M(0) + 2n\frac{\overline{m^2}}{r^3} + \bar{p}\varphi\eta\omega, \tag{8}$$

where $\bar{p}$ is the average form factor depending on the number of particles in rigid chain aggregates.

The formula for F_T will be modified analogously, with $F(0) = M(0)$ having the definite significance of the limit shearing stress outside the field. It is to be noted that in a system of strongly interacting dipole particles ($m^2/r^3 \gg kT$, with k denoting the Boltzmann constant and T denoting the temperature) there can be no regular distribution of particles in a suspension [19] or sol [20]. In this case

$$F(0) \simeq 2I_s^2\left(\frac{\bar{r}}{r}\right)^3. \tag{9}$$

Here $I_S = mn$ and $(\bar{r})^3 = n^{-1}$ is the average volume per particle. When $\bar{r} = r$ (uniform distribution of particles over the volume), then (9) becomes the well-known Volarovich–Gutkin equation [19]. Equation (9) agrees sufficiently well with the results of measurements (see Fig. 3). The noticeable difference between quantities M_T and F_T must be attributed to the conditions under which F_T was measured (in a Couette viscometer with a radial magnetic field); namely on the suspension acted some gradient of the field intensity which, apparently, compressed the layer of particles in the viscometer gap while weakening their coupling to the outer active instrument surface.

From the experiment in a rotating field with the magnetorheological effect one can determine the equilibrium distance r between particles in chains, or the dimensionless distance $r/\bar{r}$. From the data in Fig. 3 we have

$$\left(\frac{r}{\bar{r}}\right)^3 \equiv nr^3 \equiv \frac{2d(I^2)}{dM} = 0.167.$$

If the radius of a particle is much larger than the radius of action of superficial repulsion forces, then $r \simeq 2a$ and $nr^2 = 2\varphi$. In a barium hexaferrite suspension $\varphi = 0.115$, i.e., 2φ approaches the value found from measurements of the friction torque. Thus, with the relations derived here one can quite reliably estimate the range of action of superficial repulsion forces between particles. At a certain distance ($r \simeq 2a$), the magnitude of the induced or the rigid electric dipole of particles can be determined from the electrorheological effect.

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